

Metastable Vacua in Brane Worlds

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ABSTRACT: We analyze vacuum decay in brane world setups, where a free scalar field in five dimensions has a localized potential admitting metastable vacua. We study in particular the bounce solution and its properties in flat and warped spaces. In the latter case, placing into a deeply warped region the term in the potential that lifts the vacuum degeneracy, can increase indefinitely the lifetime of the false vacuum. We discuss the application to metastable vacua in supersymmetric brane-world constructions.

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1. Introduction and conclusions

The fate of metastable vacua in field theory [1, 2] is of great interest in cosmology and particle physics. The dynamics of their quantum decay toward the true vacuum rely on the knowledge of the classical Euclidean “bounce” solution. The study of finite energy soliton solutions in Minkowski space and of finite action solutions in Euclidean space is therefore crucial for the study of metastable vacua. The purpose of this paper is to study the generalization of this problem in the case of vacua generated by a scalar field living in a higher-dimensional spacetime, with a scalar potential localized in four dimensions. The field is therefore free in the bulk, with the scalar potential generating non-trivial boundary conditions. One of our main motivation for studying the case of a boundary potential is the generalization to supersymmetric theories with metastable vacua [3, 4]. In this case, constraints coming from higher-dimensional supersymmetry are such that it is much easier to construct models with localized (as opposed to bulk) superpotentials.

The search for classical solutions in this case turns out to be very interesting and rich. A first question is the dependence of the classical solution on the geometry of the internal space, flat or warped, and on its size. In the case of the bounce, we would like to understand the dependence of the width of the wall in the thin wall approximation on the extra

dimension. Moreover, since all nontrivial dynamics is encoded in boundary conditions, it suggests that the problem could be tractable to some extent even in the case where the internal space is warped. In this last case, there are several interesting questions arising. First of all, even if we are in a regime in which the 4d effective theory is valid, i.e. there is a mode much lighter than the KK masses, it is possible that the barrier separating the false from the true minimum is much higher than the mass of the lowest-lying KK states. In this case, despite the validity of the 4d effective action describing the lightest mode, there is no a priori reason why the classical solution should be the standard 4d one. Secondly, it is reasonable to expect that by placing the term lifting the degeneracy between the true and the false vacuum into a deeply warped region, it will be redshifted to small values, thus increasing indefinitely the lifetime of the false vacuum. If this were indeed possible, there would be no practical difference between living in the true vacuum or the false vacuum ! In this paper, we will be able to answer some of these questions, whereas other questions will be addressed only partially and will need future work for a complete understanding.

The plan of the paper is as follows. In Section 2 we discuss vacuum decay in a toy-model consisting of a single scalar field in a spacetime with one flat, compact extra-dimension. We show that when there is a light (compared to the KK scale) mode, the effective theory is precisely the one which admits a standard kink solution. We then work out the 5d analog of the classical 4d field equation. This can be written as a 4d differential equation, which allows a systematic calculation of the corrections to the 4d kink solution. The equation involves a differential operator containing higher derivative terms. We work out the size of the kink in the large radius limit, show that the kink become broader with increasing the radius and study its behavior near the origin. We use this to write the equation defining the euclidian bounce and check the validity of the thin wall approximation, finding that it gets worse in the 5d limit.

In Section 3 we perform the same analysis including the effect of the warping of the extra-dimension. We find that the 4d limit and the thin-wall approximation become very accurate because of the warping. However Coleman-de Luccia gravitational effects can become important and even completely lock the decay of the false vacuum.

In Section 4 we extend these considerations to the supersymmetric case. Motivated by the D3/D7 brane realizations of the ISS model, we discuss the AdS_5 version of the ISS model [3], with ISS gauge group and quarks living on the UV boundary and the mesons living in the 5d bulk. The meson-quark coupling is then localized on the UV brane, whereas the mesonic linear term in superpotential is put on the IR boundary. We find that, due to the warping, the $(\text{mass})^2$ parameter is naturally redshifted to small values, whereas metastable supersymmetry breaking becomes a non-local (in the extra dimension) effect. This has again the net effect of increasing correspondingly the lifetime of the metastable vacuum. We also analyze briefly the case where the whole superpotential is localized on the UV brane and a light mode is achieved by adding a bulk mass term for the mesons hypermultiplet. In this case mass scales are redshifted again due to a different effect, the value of the mesonic wave function on the UV brane. Both examples have a natural 4d holographic interpretation via the AdS/CFT correspondence.

Some technical details of the computations are left to three Appendices.

2. Flat 4+1 Dimensions

In this section we consider a massless scalar field in a 4+1 dimensional flat spacetime¹ in which the 5th direction (labeled by the coordinate y) extends between two rigid branes at $y = 0$ and $y = \pi R$. The bulk action is that of a free massless field, all nontrivial potential terms appearing on the boundaries² :

$$S = -\frac{1}{2} \int d^4x dy \partial_A \Phi \partial^A \Phi - \int d^4x V_0(\Phi)|_{y=0} + \int d^4x V_1(\Phi)|_{y=\pi R}. \quad (2.1)$$

The field equations and boundary conditions read:

$$\partial_y^2 \Phi + \partial^\mu \partial_\mu \Phi = 0, \quad (2.2)$$

$$\partial_y \Phi|_{y=0} = \frac{\partial V_0}{\partial \Phi}, \quad (2.3)$$

$$\partial_y \Phi|_{y=\pi R} = \frac{\partial V_1}{\partial \Phi}. \quad (2.4)$$

2.1 The Kink

Consider the situation where the brane potentials are given by:

$$V_0(\Phi) = \frac{\lambda}{4} (\Phi^2 - v^2)^2, \quad V_1(\Phi) = 0 \quad (2.5)$$

From eqs. (2.2-2.4) we see immediately that there are two “vacuum” solutions $\Phi_\pm(x, y) = \pm v$. One can ask whether there exist a solution interpolating between the two vacua, analogous to the four-dimensional domain wall (kink) that one finds with the same quartic potential (see Appendix A).

Notice that, since Φ is canonically normalized in 5D, and has mass dimension 3/2, the parameters in (2.5) have unusual mass dimensions:

$$[\lambda] = M^{-2}, \quad [v] = M^{3/2}. \quad (2.6)$$

2.1.1 Effective 4D theory

A kink-like solution is expected to exist at least in a certain region of parameter space, where one can give a four-dimensional effective description of the model. To see this, consider the linearized fluctuations around one of the two vacua (say Φ_-):

$$\Phi(x, y) = -v + \delta\Phi(x, y). \quad (2.7)$$

Decomposing the solution in eigenstates of the 4D D’Alambertian, $\delta\Phi(x, y) = \phi(y)\chi(x)$, $\square_4\chi(y) = m^2\chi(y)$, the mass spectrum is obtained by linearizing the boundary conditions (2.3-2.4):

$$[\partial_y \Phi = \mu_0^2 \Phi]_{y=0}, \quad [\partial_y \Phi = 0]_{y=\pi R} \quad (2.8)$$

¹We use signature $(-+++)$. Throughout the paper we consistently neglect the backreaction of the scalar field on the geometry [5, 6]

²The case of the bulk potential and the kink solution in the extra coordinate did lead historically to the first brane world proposals [7]. For bounce solutions for brane localized fields, see e.g. [8].

where $\mu_0^2 = 2\lambda v^2$. The mass eigenstates are the solutions of the equation:

$$m \tan m\pi R = \mu_0^2 \quad (2.9)$$

and the profile wave-function for a given mode of mass m is:

$$\phi_m(y) = \cos[m(y - \pi R)]. \quad (2.10)$$

We have a low-energy, 4D effective theory for the lowest-lying mode (of mass m_0) if $m_0 R \ll 1$. This description is valid for energies much smaller than the mass of the next KK mode, which is of order $1/R$. Under this conditions we can expand the tangent in eq. (2.9) and obtain:

$$m_0^2 \simeq \frac{\mu_0^2}{\pi R} = \frac{2\lambda v^2}{\pi R} \quad (2.11)$$

and the condition for the existence of a 4D description reads, in terms of the original parameters of the model:

$$2R\lambda v^2 \ll \pi. \quad (2.12)$$

Under these condition, inserting $\Phi(x, y) = -v + \phi_0(y)\chi_0(x)$ in the original action and integrating over y , we obtain the low-energy 4D effective action for the lowest-lying mode $\chi_0(x)$. After some integration by parts and using the bulk field equation we obtain:

$$S_{eff} = \int d^4x \left[-\frac{1}{2} \partial_\mu \chi_0 \partial^\mu \chi_0 - V_{eff}(\chi_0) \right] \quad (2.13)$$

where the effective potential is:

$$V_{eff} = \frac{m_0^2}{2} \chi_0^2 - \frac{g}{3} \chi_0^3 + \frac{h}{4} \chi_0^4; \quad (2.14)$$

$$m_0^2 \simeq \frac{2\lambda v^2}{\pi R}, \quad g = \frac{3\lambda v}{(\pi R)^{3/2}}, \quad h = \frac{\lambda}{(\pi R)^2}. \quad (2.15)$$

The extra factors of πR in the effective parameters come from the normalized wave-function profile $\phi_0(y) = (1/\sqrt{\pi R}) \cos m_0(y - \pi R)$ evaluated in $y = 0$. It is easy to check that the potential (2.14) has two zero-energy minima at $\chi_0 = 0, 2v\sqrt{\pi R}$ and a maximum at $\chi_0 = v\sqrt{\pi R}$ with $V(v\sqrt{\pi R}) = \lambda v^4/4$. In terms of the original field $\Phi = -v + \phi_0\chi_0$ these correspond exactly to the original two minima at $\Phi = \pm v$ and maximum at $\Phi = 0$.

Due to the standard double-well form of the effective potential the field equation derived from the effective action (2.13),

$$\partial_x^2 \chi(x) = \frac{\partial V_{eff}}{\partial \chi} \quad (2.16)$$

admits a kink solutions interpolating between the two vacua $\chi_0 = 0$ and $\chi_0 = 2v\sqrt{\pi R}$, which according to eqs. (A.2, A.3) has the form:

$$\chi_{kink}(x) = v\sqrt{\pi R} (1 + \tanh \mu x), \quad \mu^2 = \frac{h}{2} (v\sqrt{\pi R})^2 = \frac{\mu_0^2}{4\pi R} = \frac{\lambda v^2}{2\pi R}. \quad (2.17)$$

The kink energy density is of the order of the height of the potential barrier, λv^4 . In order for the solution we found to be reliable, this energy density must be below the KK

scale, we thus have the additional requirement $\lambda v^4 \ll 1/R^4$. This, together with (2.12), sets the range of validity of the kink solution we found. A sufficient condition is :

$$v \ll R^{-3/2}, \quad \lambda \ll R^2. \quad (2.18)$$

Although the energy density of the domain wall is R -independent, the integrated total energy is not:

$$E = \int_{-\infty}^{+\infty} dx \left(\frac{d\chi_{kink}}{dx} \right)^2 \sim v^2 \pi R \mu = \sqrt{\lambda \pi R} v^3 \quad (2.19)$$

2.1.2 The 5D equation

The argument of the previous subsection suggest that a kink solution to the model (2.1) should exists, at least in the range of parameters satisfying (2.18). Now we want to look for similar solutions from a purely 5D perspective, without having to rely on the 4D effective theory approach.

Let us return to eqs. (2.2-2.4). We look for solutions depending on y and one of the Minkowski coordinates (say x). The most general (real) solution to (2.2) can be written as:

$$\Phi(x, y) = g(x + iy) + (g(x + iy))^*. \quad (2.20)$$

The boundary condition at $y = \pi R$ tells us that

$$Im [g'(x + i\pi R)] = 0, \quad (2.21)$$

where a prime denotes derivative w.r.t. the argument. This equation is satisfied if $F(x) = g(x + i\pi R)$ is a real function³. This also implies that, for z complex, $(F(z))^* = F(z^*)$.

Next, consider the boundary conditions at $y = 0$. Defining $f(x) = \Phi(x, 0)$, $h(x) = \partial_y \Phi(x, 0)$, eq. (2.3) reads:

$$h(x) = \lambda f(x) \left[(f(x))^2 - v^2 \right]. \quad (2.22)$$

Formally, we can write:

$$\begin{aligned} f(x) &= g(x) + g(x)^* = F(x - i\pi R) + F(x + i\pi R) \\ &= \left(\exp[-i\pi R \partial_x] + \exp[i\pi R \partial_x] \right) F(x), \end{aligned} \quad (2.23)$$

$$\begin{aligned} h(x) &= ig'(x) - ig'(x)^* = iF'(x - i\pi R) - iF'(x + i\pi R) \\ &= i \left(\exp[-i\pi R \partial_x] - \exp[i\pi R \partial_x] \right) \partial_x F(x) \\ &= \tan(\pi R \partial_x) \partial_x f(x). \end{aligned} \quad (2.24)$$

³That is, $g(z)$ has an expansion of the form

$$g(z) = \sum c_n (z - i\pi R)^n$$

with real coefficients c_n .

Using the last line in eq. (2.24) we arrive at a closed equation⁴ for $f(x)$:

$$\tan(\pi R \partial_x) \partial_x f = \lambda f [f^2 - v^2] . \quad (2.26)$$

Another, maybe less general way, in order to arrive at (2.26) is to start from the bulk solution

$$\Phi(x, y) = \int dp a_p e^{px} \cos(py + \alpha_p) , \quad (2.27)$$

where a_p (α_p) are arbitrary coefficients (phases). Boundary conditions at $y = \pi R$ fixes $\alpha_p = -p \pi R$, whereas boundary conditions in $y = 0$ gives by a straightforward computation (2.26), by using the replacement $p \rightarrow \partial_x$. This method will generalize in a straightforward manner to the warped case discussed in the next section.

A solution to (2.26) gives $f(x) = \Phi(x, 0)$, which can then be extended into the bulk to a full solution:

$$\begin{aligned} \Phi(x, y) &= \text{Re}[f(x + iy)] + \tan(\pi R \partial_x) \text{Im}[f(x + iy)] \\ &= [\cos y \partial_x + (\tan \pi R \partial_x)(\sin y \partial_x)] f(x) . \end{aligned} \quad (2.28)$$

Eq. (2.26) has various interesting properties. It should be understood as a series in derivatives of increasing order. If we take $f(x)$ to be a 4D mass eigenstate, $f(x) = e^{mx}$, and linearize the r.h.s, we get back to the eigenvalue equation (2.9). So the information about the spectrum of the model is contained in (2.26).

Now suppose that we can keep the lowest order in the expansion of the l.h.s. (for any given solution we can later check whether this approximation is justified). We get a second order equation for f which looks exactly as the one for the 4D kink:

$$\partial_x^2 f = \frac{\lambda}{\pi R} f [f^2 - v^2] \quad (2.29)$$

whose solution is again given by eq. (A.3):

$$f(x) = v \tanh \mu x, \quad \mu^2 = \lambda v^2 / (2\pi R). \quad (2.30)$$

Notice that the characteristic scale μ is the same as in eq. (2.17).

We can extract considerable information from eq. (2.26) even when the 4D limit does not hold. Consider a solution $f(x)$ that approaches $\pm v$ as $x \rightarrow \pm\infty$. We can estimate the width of the kink by expanding $f(x) = -v + \eta(x)$ and solving the linear equation for η

⁴As for the standard kink, this equation can be obtained from a the point-particle analog model, with potential $-V(f)$ and an exotic kinetic term:

$$S = \int dx \left[V(f) - \frac{1}{2} f \tan(\pi R \partial_x) \partial_x f \right] , \quad (2.25)$$

which is the same as the “effective action” whose variation gives (2.26).

in the asymptotic large $|x|$ region: assuming $\eta(x) \sim e^{-|x|/l_w}$, where l_w is a measure of the wall width, we find:

$$\frac{1}{l_w} \tan\left(\frac{\pi R}{l_w}\right) = 2\lambda v^2. \quad (2.31)$$

In the 4D limit we get the expected result, namely $l_w = 1/\mu$, with μ as in (2.30). In any case, the size of the wall cannot exceed $2R$, and this value is approached in the opposite limit, when $R\lambda v^2 \gg 1$.

Another interesting length scale is the one corresponding to the regime of the validity of the linear slope of the solution $f(x) \sim (1/l_0)x$, when the variation (derivative) of the field f is maximal⁵. This can be estimated by linearizing eq. (2.26) around $f = 0$. Setting $f(x) = \eta \sin(x/l_0)$, with η a constant, we find:

$$\frac{1}{l_0} \tanh\left(\frac{\pi R}{l_0}\right) = \lambda v^2. \quad (2.32)$$

In the 4d limit we get as expected $l_0 \sim 1/\mu$, whereas in the 5d regime we get $l_0 \sim (1/\lambda v^2)$. In the 5d limit therefore, the size of the kink becomes larger and larger, whereas large variations of the field are confined into a fixed region.

We can put eq. (2.26) in integral form. Going in Fourier space, and using the identity

$$\int dk \frac{e^{ikx}}{k \tanh k} = -\log[\sinh |x/2|], \quad (2.33)$$

and $f(0) = 0$, we obtain:

$$\hat{f}(u) = \frac{1}{2\pi} \int dt \log \left[\sinh \left| \frac{u-t}{2R\lambda v^2} \right| \right] \hat{f}(t)(\hat{f}(t)^2 - 1) \quad (2.34)$$

where we have defined $\hat{f}(u) \equiv v^{-1}f(u/(\lambda v^2))$. Since the kernel in (2.34) behaves as $|x-t|$ for large t , $f(t)$ must necessarily approach one of the extrema $f = 0, \pm v$ as $t \rightarrow \pm\infty$.

It is also of interest to find the generalization of equ. (A.4) which expresses the vanishing of the "energy" of the point particle analog system. The determination of this quantity turns out to be non-trivial. The details of the calculation are relegated to the Appendix where we show that the kink verifies the following equation

$$\mathcal{E} = \frac{1}{2} \frac{\tan(\pi R \partial)}{\pi R \partial} [(\partial f)^2 - (\tan(\pi R \partial) \partial f)^2] - \frac{1}{\pi R} V(f) = 0. \quad (2.35)$$

The leading terms in the expansion in powers of R are

$$\frac{1}{2}(\partial f)^2 - \frac{1}{\pi R} V(f) + \dots = 0. \quad (2.36)$$

They reproduce the 4D equation (A.4). The first order correction to the 4D equation of motion can also be simply obtained:

$$\frac{1}{2}[(\partial f)^2 + \frac{1}{3}(\pi R)^2(2\partial^3 f \partial f - (\partial^2 f)^2)] - \frac{1}{\pi R} V(f) = 0. \quad (2.37)$$

⁵In the usual $d+1$ kink solution (A.3) both l_w and l_0 are of order $1/\mu$.

If we expand f in powers of R and write $f = f_0 + (\pi R)^2 f_2 + \dots$, then f_2 can be determined from the first order equation:

$$f'_0 f'_2 - \frac{1}{\pi R} V'(f_0) f_2 + \frac{1}{6} (2\partial^3 f_0 \partial f_0 - (\partial^2 f_0)^2) = 0, \quad (2.38)$$

where f_0 is the 4D solution $v \tanh \mu x$. Using the zeroth order equation of motion $f''_0 = \frac{1}{\pi R} V'(f_0)$ the solution can be written as

$$f_2(x) = \frac{f'_0(x)}{6} \int_0^x du \frac{((\partial^2 f_0)^2 - 2\partial^3 f_0 \partial f_0)}{(\partial f_0)^2}, \quad (2.39)$$

where the integration constant was fixed by requiring f_2 to be odd. Finally the integration can be done to yield

$$f_2(x) = \frac{1}{6} f'_0(x) \left[-\frac{2\lambda}{\pi R} v^2 x + 4\sqrt{\frac{2\lambda}{\pi R}} f_0(x) \right]. \quad (2.40)$$

Explicitly, the solution to the first nontrivial order reads

$$f(x) = v \tanh(\mu x) \left[1 - \frac{(2\pi R \mu)^2}{6 \cosh^2(\mu x)} \left(\frac{\mu x}{\tanh(\mu x)} - 2 \right) \right]. \quad (2.41)$$

This shows that the zeroth order approximation is valid as long as $(2\pi R \mu)^2$ is much smaller than one.

The identification of the kink as a solution of the equation $\mathcal{E} = 0$, has an important consequence: such solutions can never cross the lines $f = \pm v$. This is analog to the usual two-derivative kink: there, eq. (A.4) implies that $f = \pm v$ are the fixed points for the first order flow of the quantity Φ , and as such they cannot be crossed in finite “time.” The corresponding statement in the case of eq. (2.35) is proven in Appendix (C). As an important consequence of this fact, the (true) energy of any solution interpolating between $+v$ and $-v$ is always positive, as we will see in the next subsection.

Another useful form of \mathcal{E} is obtained by using the equations of motion (2.26) in (2.35) to put it in the form

$$\mathcal{E} = \frac{1}{2} \frac{\tan(\pi R \partial)}{\pi R \partial} [(\partial f)^2 - (V'(f))^2] - \frac{1}{\pi R} V(f) = 0. \quad (2.42)$$

2.2 The Bounce

Next, we add a linear potential on the brane at $y = \pi R$, of the form

$$V_1 = b(\Phi - v), \quad [b] = M^{5/2}. \quad (2.43)$$

In this case, eq. (2.26) becomes:

$$\tan(\pi R \partial_x) \partial_x f - b = \lambda f [f^2 - v^2]. \quad (2.44)$$

The addition of (2.43) breaks the $\Phi \rightarrow -\Phi$ symmetry and lifts the degeneracy between the two vacua, making one of them metastable. We will follow Coleman [1] and estimate

the decay rate of the metastable vacuum. We look for a “bounce” solution $\Phi^B(y, t_E, \vec{x})$, i.e. a solution of the Euclidean field equations that interpolates between the true vacuum at small $\rho \equiv \sqrt{t_E^2 + |\vec{x}|^2}$, and the false vacuum at large ρ . The bounce must have finite action relative to the false vacuum. Then the tunneling amplitude is given by:

$$\Gamma = \exp \{ -S_E[\Phi^B] + S_E[\Phi^F] \}. \quad (2.45)$$

Here $S_E[\Phi]$ is the euclidean version of the action (2.1) evaluated on the field configuration Φ^B (the bounce) and Φ^F (the false vacuum).

We look for a bounce solution with $O(4)$ -symmetry, i.e. depending only on y and on the Euclidean radial coordinate ρ . The euclidean field equation in these coordinates reads:

$$\partial_y^2 \Phi + \partial_\rho^2 \Phi + \frac{3}{\rho} \partial_\rho \Phi = 0 \quad (2.46)$$

$$\partial_y \Phi|_{y=0} = \lambda \Phi (\Phi^2 - v^2), \quad (2.47)$$

$$\partial_y \Phi|_{y=\pi R} = b. \quad (2.48)$$

and we look for a solution that approaches the true vacuum Φ^T at $\rho \simeq 0$ and the false vacuum Φ^F at $\rho \simeq \infty$. Following Coleman, we consider the symmetry breaking term as a perturbation: we approximate both the true and the false vacuum to be the same as the unperturbed ones ($\Phi(x, y) = \pm v$), and moreover we set $b = 0$ when solving the field equation. As a further approximation, we assume we are in the “thin wall” limit, in which we can neglect the last term in eq. (2.46). This is justified when the transition between the true and false vacuum takes place in a small region of width l_b around a radius $\rho_0 \gg l_b$. Under these assumptions, the problem reduces to the one of the previous section, i.e. finding a domain wall solution centered around ρ_0 :

$$\Phi^B(\rho, y) = \begin{cases} -v & 0 < \rho \ll \rho_0, \\ \Phi_{kink}(\rho - \rho_0, y) & \rho \simeq \rho_0 \\ v & \rho \gg \rho_0, \end{cases} \quad (2.49)$$

Requiring that the bounce has minimal action provides a variational problem for the parameter ρ_0 . The bounce action is:

$$\frac{S_b}{2\pi^2} = \int_0^{\pi R} dy \int d\rho \rho^3 \frac{1}{2} [(\partial_\rho \Phi^B)^2 + (\partial_y \Phi^B)^2] - \int d\rho \rho^3 (V_0(\Phi^B(\rho, 0)) - V_1(\Phi^B(\rho, \pi R))). \quad (2.50)$$

Integrating by parts the bulk piece and using the field equations the above expression reduces to boundary terms. Approximating the solution as in (2.49), we obtain:

$$\frac{S_b}{2\pi^2} \approx -2bv \frac{\rho_0^4}{4} + \rho_0^3 S_{wall}. \quad (2.51)$$

Here, S_{wall} is the energy stored in the wall. In our approximation can be thought as concentrated in a small region around ρ_0 , and can be approximated by the total energy of the kink, i.e. the solution of eq. (2.26) :

$$S_{wall} \simeq \int_{-\infty}^{+\infty} dx \left(V(\Phi) - \frac{1}{2} \Phi \frac{dV}{d\Phi} \right)_{y=0} = \int_{-\infty}^{+\infty} dx \frac{\lambda}{4} (v^4 - f_{kink}^4(x)). \quad (2.52)$$

In the 4D regime, in which eqs. (2.29) and (2.30) hold, then after an integration by parts we obtain:

$$S_{wall} = \pi R \int_{-\infty}^{+\infty} dx (\partial_x f_{kink})^2 = \frac{4}{3} \pi R \mu v^2 = \frac{4}{3} \sqrt{\pi R \lambda} v^3 \quad (2.53)$$

Using this result in eq. (2.51) and minimizing the action with respect to ρ_0 we find:

$$\rho_0 \sim \frac{S_{wall}}{bv} \sim \sqrt{\pi R \lambda} \frac{v^2}{b} \quad (2.54)$$

and the thin wall approximation holds if

$$1 \ll \mu \rho_0 = \frac{\lambda v^3}{b} \quad (2.55)$$

which is the same condition [1] finds in the purely 4D case, and that we would have obtained had we started from the 4D effective action in Section 2.1.1.

From the previous discussion, it is clear that the thin-wall approximation gets worse and worse as we move away from the 4D regime, i.e. as $R\lambda v^2$ becomes large. In fact, as discussed earlier, from eqs. (2.31) and (2.32) it follows that for $R\lambda v^2 \gg 1$ the width of the wall becomes much larger than the size of the region where the field profile has its largest variation (i.e. close to the $f = 0$), therefore the wall energy density gets spread over a larger and larger region.

3. The warped case

We are now going to repeat the steps in the previous section in a slice of AdS_5 bounded by two branes at $y = 0, \pi R$. We parametrize the metric as:

$$ds^2 = dy^2 + e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu . \quad (3.1)$$

We will always assume a large warping, $\exp[k\pi R] \gg 1$.

The field equation is

$$\partial_y^2 \Phi - 4k \partial_y \Phi + e^{2ky} \partial_\mu^2 \Phi = 0 . \quad (3.2)$$

with the same boundary conditions as before. The general solution for a mass eigenstate, $\square_4 \Phi = m^2 \Phi$, has the form:

$$\Phi_m(y) = e^{2ky} B_2 \left(\frac{m}{k} e^{ky} \right) \quad (3.3)$$

where $B_\nu(x) = a_m J_\nu(x) + b_m N_\nu(x)$ is an appropriate combination of Bessel functions, whose coefficients are to be determined along with the mass eigenvalues m , from the boundary conditions (2.3-2.4). The correct linear combination is :

$$B_\nu(x) = J_\nu(x) - \frac{J_1(m e^{k\pi R}/k)}{N_1(m e^{k\pi R}/k)} N_\nu(x), \quad (3.4)$$

and the resulting equation for the KK masses is:

$$m \frac{B_1(m/k)}{B_2(m/k)} = \mu_0^2 . \quad (3.5)$$

Typically the lowest KK mass is of order:

$$m_{kk} \sim k e^{-k\pi R}. \quad (3.6)$$

If we are in the 4D regime, when the lowest mode has mass $m_0 \ll m_{kk}$, expanding the Bessels in (3.5) we find:

$$m_0^2 \sim \frac{2k\mu_0^2}{(1 - e^{-2k\pi R})} \quad (3.7)$$

so the 4D regime demands that

$$\lambda v^2 \ll k e^{-2k\pi R}. \quad (3.8)$$

In this 4d regime, the wave function of the zero mode is basically flat, whereas the $\Phi \rightarrow -\Phi$ symmetry breaking term is redshifted by the warp factor $b \rightarrow b \exp(-4k\pi R)$. The 4d Coleman expression for the bounce action is therefore valid and produces a huge enhancement of the lifetime of the false vacuum compared to the unwarped case. This is one of the main advantages in constructing metastable vacua in warped spaces.

However, it would be very interesting to also understand the opposite regime, namely when $\mu_0^2 = \lambda v^2$ is much larger than the KK scale. Notice that in this regime the mass eigenstates are approximately given by the solutions of $B_2(m/k) = 0$, since in this case the l.h.s. of eq. (3.5) is large.

Let us now look for a kink-like solution, depending on the coordinates x and y . From the flat case, we learned how to read-off an effective one-dimensional equation for the field at $y = 0$ from the spectral equation. Repeating the argument that leads to (2.26), starting from the general bulk solution with correct boundary condition at $y = \pi R$

$$\begin{aligned} \Phi(x, y) &= e^{2ky} \int dp a_p e^{px} \left[J_2\left(\frac{p}{k} e^{ky}\right) - \frac{J_1\left(\frac{p}{k} e^{k\pi R}\right)}{N_1\left(\frac{p}{k} e^{k\pi R}\right)} N_2\left(\frac{p}{k} e^{ky}\right) \right] \\ &= e^{2ky} \frac{J_2\left(\frac{\partial_x}{k} e^{ky}\right) - \frac{J_1\left(\frac{\partial_x}{k} e^{k\pi R}\right)}{N_1\left(\frac{\partial_x}{k} e^{k\pi R}\right)} N_2\left(\frac{\partial_x}{k} e^{ky}\right)}{J_2\left(\frac{\partial_x}{k}\right) - \frac{J_1\left(\frac{\partial_x}{k} e^{k\pi R}\right)}{N_1\left(\frac{\partial_x}{k} e^{k\pi R}\right)} N_2\left(\frac{\partial_x}{k}\right)} \Phi(x, 0), \end{aligned} \quad (3.9)$$

we obtain:

$$\frac{B_1(m/k)}{B_2(m/k)} \Big|_{m \rightarrow \partial_x} \partial_x f = \lambda f [f^2 - v^2], \quad (3.10)$$

where $f(x) \equiv \Phi(x, 0)$.

In the 4D limit we can take the first term in the expansion of the l.h.s. of eq. (3.10), and we find again a second order equation, of the form:

$$\partial_x^2 f = \frac{2k\lambda}{(1 - e^{-2k\pi R})} f [f^2 - v^2] \quad (3.11)$$

which is the usual kink equation.

Let us estimate the width of the kink in the opposite regime, $\lambda v^2 \gg m_{kk}^2/k$. Using the same argument as in the previous section, and writing $f(x) = \pm v + \eta e^{\pm x/l_w}$ we find that l_w obeys:

$$l_w^{-1} \frac{B_1(1/(kl_w))}{B_2(1/(kl_w))} = \mu_0^2, \quad (3.12)$$

which comparing with eq. (3.5) means that the maximal width is equal to the inverse mass of the lowest KK mode, i.e. of order (3.6).

Let us now add the linear term (2.43) on the IR brane, as we did in the flat case. Making the same approximations as in Section 2.2 (treat V_1 as a perturbation, and use the thin-wall approximation), we arrive at the following bounce action:

$$\begin{aligned} \frac{S_b}{2\pi^2} &= \int_0^{\pi R} dy e^{-4ky} \int d\rho \rho^3 \frac{1}{2} \left[e^{2ky} (\partial_\rho \Phi^B)^2 + (\partial_y \Phi^B)^2 \right] \\ &\quad - \int d\rho \rho^3 \left(V_0(\Phi^B(\rho, 0)) - e^{-4k\pi R} b(\Phi^B(\rho, \pi R) - v) \right) \\ &\approx -2bv \frac{\rho_0^4}{4} e^{-4k\pi R} + \rho_0^3 S_{wall}, \end{aligned} \quad (3.13)$$

where again we have assumed that $\Phi^B(\rho, y) = \Phi^T(\rho, y) = -v$ for $\rho < \rho_0$, $\Phi^B(\rho, y) = \Phi^F(\rho, y) = +v$ for $\rho > \rho_0$, and $\Phi^B(\rho, y) = \Phi_{kink}$ for $\rho \approx r_0$. Notice the appearance of the warp-factor in the first term of eq. (3.13). S_{wall} is the same as in eq. (2.52), and it is localized on the brane at $y = 0$ (we are neglecting the subleading contribution from the IR brane to the wall energy).

Minimizing eq. (3.13) with respect to ρ_0 we find:

$$\rho_0 \sim \frac{S_{wall}}{bv} e^{4k\pi R} \quad (3.14)$$

and if S_{wall} is not too small this leads to an exponentially large radius of the vacuum bubble, and hence an exponentially small decay rate.

We can give a crude estimate of S_{wall} as follows. Assume that $f_{kink}(x)$ can be approximated piece-wise as:

$$f_{kink}(x) \approx \begin{cases} -v & x < -l_w, \\ \frac{v}{l_w} x & -l_w < x \simeq l_w \\ v & x > l_w \end{cases} \quad (3.15)$$

Then evaluating the l.h.s. of (2.52) with this approximation we find

$$S_{wall} \sim l_w \lambda v^4 \simeq \frac{\lambda v^4}{k} e^{k\pi R} \quad (3.16)$$

In practice this may be an overestimate, since the linear regime assumed in (3.15) may not be valid for the whole width of the wall. But we can say that S_{wall} is larger than just the contribution from the linear region:

$$S_{wall} > S_{min} \approx l_{lin} \lambda v^4 \quad (3.17)$$

where l_{lin} is the region around the origin where the linear approximation (3.15) is justified. It seems reasonable to believe that this region is independent of R for large enough R . In the flat case this region is of the order $\mu_0^2 = \lambda v^2$ for large μ_0 . We can repeat the same

analysis of Section (2.1.2) to estimate the slope of the solution near $x = 0$. Assuming a behavior of the type $f(x) \sim \sin(x/l_0)$ we get the following equation:

$$\frac{i}{l_0} \frac{J_1(\frac{i}{kl_0}) - \frac{J_1(\frac{ie^{k\pi R}}{kl_0})}{N_1(\frac{ie^{k\pi R}}{kl_0})} N_1(\frac{i}{kl_0})}{J_2(\frac{i}{kl_0}) - \frac{J_2(\frac{ie^{k\pi R}}{kl_0})}{N_1(\frac{ie^{k\pi R}}{kl_0})} N_2(\frac{i}{kl_0})} = -\mu_0^2 \quad (3.18)$$

Now, let us assume that $kl_0 \gg 1$, so we can expand the Bessel functions evaluated in $(kl_0)^{-1}$ (but not the ones evaluated in $(kl_0)^{-1}e^{k\pi R}$). The quantity $J_1(ie^{k\pi R}/kl_0)/N_1(ie^{k\pi R}/kl_0)$ is never small, since $J_1(ix)$ has no zeros outside the origin. Using this fact, and expanding the Bessel functions of argument $(kl_0)^{-1}$ *only*, we obtain to lowest order:

$$\frac{1}{kl_0^2} \simeq 2\lambda v^2. \quad (3.19)$$

This result was obtained under the assumption $kl_0 \gg 1$, therefore it is valid if $\lambda v^2 \ll k$. This assumption is needed anyway, since we are in curved space, with a curvature scale of order k , and we are neglecting the backreaction of the scalar field Φ on the background, as well as the contribution of $V(\Phi)$ to the brane stress tensor.

Using l_0 from (3.19) as an estimate of the width of the linear region, we get a more conservative lower bound on S_{wall} from (3.17):

$$S_{wall} \gtrsim l_0 \lambda v^4 = \sqrt{\frac{\lambda}{k}} v^3 \quad (3.20)$$

up to $O(1)$ coefficients. If this is the case the size of the bounce is:

$$\rho_0 \gtrsim \sqrt{\frac{\lambda}{k}} \frac{v^2}{b} \exp[4k\pi R] \quad (3.21)$$

and the thin-wall approximation holds if

$$\rho_0/l_w \sim \frac{v^2 \sqrt{k\lambda}}{b} \exp[3k\pi R] \gg 1, \quad (3.22)$$

which is easily satisfied due to the warp-factor. The decay rate is also exponentially suppressed: plugging (3.21) into (3.13) we obtain:

$$S_b \simeq \frac{\lambda^2 v^9}{k^2 b^3} \exp[12k\pi R] \quad (3.23)$$

which gives a huge lifetime $\tau = e^{S_b}$ even for moderate warping. This estimate shows that, in the warped case, we don't need to restrict to the 4-dimensional regime⁶ in order to have a small vacuum decay rate.

⁶As this regime demands that $\lambda v^2 \ll k \exp[-2k\pi R]$, this would impose a very strong constraint on the model parameters. See however the next section for a different model.

There is an important omission in our previous discussion, the possible gravitational effects on the creation of the bubble. Indeed, Coleman and de Luccia showed [2] in the 4d context that gravitational effects are negligible only in the case

$$\frac{\rho_0}{\Lambda_0} \ll 1 , \quad (3.24)$$

where Λ_0 is the radius such that the bubble radius equals the Schwarzschild radius. For the 4d version of our model it equals $\Lambda_0 = (16G_N b v/3)^{-1/2}$, where $G_N = 1/M_P^2$ is the 4d Newton constant. In our case and when the 4d approximation is valid, the two length scales scale with the warp factor as

$$\rho_0 \rightarrow \exp[4k\pi R] \rho_0 \quad , \quad \Lambda_0 \rightarrow \exp[2k\pi R] \Lambda_0 . \quad (3.25)$$

Then, neglecting factors of order one, gravity effects on the creation of the bubble are negligible when

$$\exp[2k\pi R] \frac{1}{M_P} \sqrt{\frac{\lambda v^5}{b}} \ll 1 . \quad (3.26)$$

If the 4d limit (3.8) is satisfied but (3.26) is violated, as shown in [2], there are two different cases. In the first, the metastable vacuum has positive energy whereas the true vacuum where we live has zero energy. Then gravity effects increase substantially the probability of tunneling. In the second case, the metastable vacuum has zero energy and tunnels into a negative energy stable vacuum. In this case, gravity effects increase the lifetime of the metastable vacuum. In the limit where $\rho_0 > 2\Lambda_0$ the bubble cannot form and the metastable vacuum becomes completely stable. This becomes therefore one important outcome of having a warped extra dimension, in the case where (3.25) is violated.

4. Supersymmetric extension : the AdS-ISS model

Recently, there was a renewed interest in metastable vacua from the point of view of supersymmetry breaking [3], with further applications to gauge mediation models [12] and moduli stabilization [13]. The proposal in [3] used the electro-magnetic Seiberg duality to argue for the existence of metastable vacua in the supersymmetric QCD with a number of flavors $N_c + 1 < N_f < 3N_c/2$. In the IR free magnetic description and before adding the effects of the (magnetic) gauge group, the model is described by the O’Raifeartaigh-type model

$$W = hq\Phi\tilde{q} - h\tilde{\mu}^2 Tr\Phi , \quad (4.1)$$

where q_i^a ($\tilde{q}_b^{\bar{j}}$) are the magnetic quarks (antiquarks), Φ_j^i are the mesons, $a, b = 1 \cdots N$ are color indices and $i, j = 1 \cdots N_f$ are flavor ones. Supersymmetry is broken by the ”rank condition”, in the sense that the supersymmetry condition

$$F_\Phi = hq\tilde{q} - h\tilde{\mu}^2 I_{N_f} , \quad (4.2)$$

where I_{N_f} is the $N_f \times N_f$ identity matrix, cannot be satisfied, since $q\tilde{q}$ is a matrix of rank at most equal to $N < N_f$. One of the important requirements for the metastable vacuum

to be long-lived in the ISS model is $\epsilon \equiv \tilde{\mu}/\Lambda_m \ll 1$, where Λ_m is the Landau pole of the magnetic theory. From a string theory viewpoint [4], one natural realization of the ISS model, in its magnetic description, is in terms of D3/D7 brane configurations, with the ISS gauge group realized on the D3 branes, with (anti) quarks coming from the D3-D7 sector and the magnetic mesons being the positions of a stack of D7 branes.

The purpose of this section is to analyze in a field-theoretical example the effect that the warping of the internal space, generated by the branes, could have on the model. We model this effect by considering a five-dimensional supersymmetric model in a slice of AdS_5 [9] with the metric

$$ds_5^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (4.3)$$

with ISS gauge fields and the quarks, antiquarks confined to the UV boundary $y = 0$ and the mesons promoted to a hypermultiplet (Φ_1, Φ_2) propagating into the 5d bulk, with Z_2 parities $(+, -)$. The mesons-quark coupling is localized on the UV brane, whereas we choose to put the linear term in the (Z_2 even) mesons Φ_1 in the superpotential on the IR brane⁷. As we will show below, due to the exponential warp factor, the mass parameter $\tilde{\mu}$ will be redshifted such that the lifetime of the metastable vacuum becomes arbitrarily large. In a manifest 4d supersymmetric language [10, 14], the Lagrangean describing the system is

$$\begin{aligned} S = \int d^4x dy \left\{ \int d^4\theta e^{-2ky} (\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2) + \int d^2\theta e^{-3ky} (\Phi_2 \partial_y \Phi_1 + \text{h.c.}) \right. \\ \left. + \left[\int d^4\theta (q^\dagger q + \tilde{q}^\dagger \tilde{q}) + \int d^2\theta (hq\Phi_1\tilde{q} + W_{np}(\Phi_1) + \text{h.c.}) \right] \delta(y) \right. \\ \left. - \left[\int d^2\theta e^{-3k\pi R} (h\tilde{\mu}^2 \Phi_1 + \text{h.c.}) \right] \delta(y - \pi R) \right\}, \quad (4.4) \end{aligned}$$

where W_{np} is the non-perturbative mesonic superpotential arising in the field direction where the mesons Φ_1 get vev's, give masses to the quarks (antiquarks) and generate the IR dynamics restoring supersymmetry. The (metastable) supersymmetry breaking becomes now a non-local effect and arises due to the impossibility, in the absence of W_{np} , to solve the supersymmetric condition:

$$e^{-2ky} F_{\Phi_1} = -\partial_y \left(e^{-3ky} \Phi_2 \right) + (hq\tilde{q} + \partial_{\Phi_1} W_{np}) \delta(y) - e^{-3k\pi R} h\tilde{\mu}^2 I_{N_f} \delta(y - \pi R). \quad (4.5)$$

In order to cancel the last term in eq. (4.5), the Z_2 -odd mesons Φ_2 acquires a non-trivial profile:

$$\Phi_2 = e^{3k(y-\pi R)} (h\tilde{\mu}^2/2) I_{N_f} \epsilon(y). \quad (4.6)$$

If $W_{np} = 0$ supersymmetry is broken: with (4.6), and using $\partial_y \epsilon(y) = 2[\delta(y) - \delta(y - \pi R)]$, eq. (4.5) becomes:

$$e^{-2ky} F_{\Phi_1} = \delta(y) \left[hq\tilde{q} - e^{-3k\pi R} h\tilde{\mu}^2 I_{N_f} \right], \quad (4.7)$$

⁷A geometrical construction in a string context, similar in spirit, was proposed in [18].

which cannot vanish due to the rank condition. Notice that the parameter which controls supersymmetry breaking is not $\tilde{\mu}$, but rather

$$\mu_{eff}^2 = e^{-3k\pi R} \tilde{\mu}^2, \text{ since } q\tilde{q} = e^{-3k\pi R} \tilde{\mu}^2 I_N. \quad (4.8)$$

The presence of W_{np} restores supersymmetry by producing sources which do add up to zero. From the point of view of the bulk fields, in the metastable vacuum Φ_1 gets a boundary mass term

$$\mu_0^2 = h^2 \langle q^\dagger q + \tilde{q}^\dagger \tilde{q} \rangle = 2h^2 N e^{-3k\pi R} \tilde{\mu}^2, \quad (4.9)$$

whereas in the supersymmetric vacuum it gets also localized nonperturbative interactions. The formally divergent terms $\delta(0)$ in (4.4) do not appear in physical quantities, as shown in various similar situations [11].

Notice the close analogy of this model with the toy model analyzed in section 3: the symmetry breaking parameter is redshifted by a power of the scale factor. However in this model the validity of the 4D limit is automatic, and does not require an additional fine tuning: the existence of a light mode for Φ_1 requires $\mu_0^2 \ll k \exp[-2k\pi R]$, and from eq. (4.9) we see that this does not impose any strong constraint on h and $\tilde{\mu}$, provided the warp factor is large. Therefore, since the 4D limit analysis holds, the smallness of the symmetry-breaking parameter due to the redshift leads immediately to an exponential enhancement of the lifetime of the metastable vacuum. There is one critical point to check: this conclusion is valid if the wave function of the lightest mode of Φ_1 does not grow too fast in the IR and destroys the redshift of the mass term $\tilde{\mu}$, transparent in (4.4). In the limit where the 4d effective theory is valid, i.e. the lightest mode is much lighter than the KK masses $m \ll k e^{-k\pi R}$, its corresponding wave function reads approximatively

$$\Phi_1^{(0)}(y) \simeq d_1 e^{4ky} \left[1 - \frac{m^2}{12k^2} e^{2ky} \right] + d_2 \left[1 + \frac{m^2}{4k^2} e^{2ky} \right]. \quad (4.10)$$

Boundary conditions determine then the mass spectrum to be given by the equation

$$-2m^2 e^{-2k\pi R} = \left(\mu_0^2 - \frac{m^2}{2k} \right) \left(4k - \frac{m^2}{2k} e^{2k\pi R} \right). \quad (4.11)$$

Due to the validity of the 4D limit, we get the 4D result (see section 3) $m^2 \simeq 2k\mu_0^2$ and a corresponding wavefunction (4.10) which is constant in y to the leading order. In this case, the redshift of the mass parameter $\tilde{\mu}^2 \rightarrow \tilde{\mu}^2 e^{-3k\pi R}$ is effective and produces a huge enhancement of the lifetime of the false vacuum. Notice that with respect to a 5d flat metric, the light mode is actually localized on the UV boundary. Since the KK modes and the linear term are localized on the IR boundary, this explains the enhancement of the lifetime of the metastable vacuum.

Another interesting case, with the same matter content, is when the whole superpotential is localized on the UV boundary. In this case, there is no redshift of the mass parameter $\tilde{\mu}$ and generically no light mode. One way to obtain a light mode even in the case $k e^{-2k\pi R} \ll \mu_0^2 \ll k$ is to add a bulk mass for the hypermultiplet, which is tuned appropriately against the boundary mass. This is a tuning in a non-supersymmetric setup, but the tuning is

actually required and protected versus radiative corrections by supersymmetry [14, 16]. In this case the bulk mass m_b and the boundary masses μ_0, μ_π for the scalar component of Φ_1 , in the false vacuum, are given by

$$\begin{aligned}\frac{m_b^2}{k^2} &= \alpha^2 - 4 = \left(c - \frac{3}{2}\right) \left(c + \frac{5}{2}\right) , \\ \mu_0^2 &= h^2 \langle q^\dagger q + \tilde{q}^\dagger \tilde{q} \rangle + \left(\frac{3}{2} - c\right) k , \\ \mu_\pi^2 &= - \left(\frac{3}{2} - c\right) k ,\end{aligned}\tag{4.12}$$

where $\alpha = |c + 1/2|$. Since we want to preserve in the first approximation the AdS_5 geometry, we are interested in small backreaction of the scalar field and therefore small bulk mass $\alpha \simeq 2$. There is one interesting example of this type, with $c = -5/2$ and therefore zero bulk mass for Φ_1 , with non-vanishing brane localized masses. In this case we find a light scalar mode localized on the IR brane, with wave-function and mass given by

$$\begin{aligned}\Phi_1^{(0)}(x, y) &\sim e^{-3k\pi R} e^{4ky} \phi(x) , \\ m^2 &\simeq 6kh^2 \langle q^\dagger q + \tilde{q}^\dagger \tilde{q} \rangle e^{-6k\pi R} .\end{aligned}\tag{4.13}$$

The term $\exp(-3k\pi R)$, important in what follows, comes from normalization of the 4d kinetic term of the light mode $\phi(x)$. The four dimensional Lagrangean in this case is very close to the 4d ISS Lagrangean. Auxiliary fields are

$$\begin{aligned}e^{-2ky} F_{\Phi_1} &= -\partial_y(e^{-3ky} \Phi_2) + (hq\tilde{q} - h\tilde{\mu}^2 + \partial_{\Phi_1} W_{np})\delta(y) , \\ F_q &= e^{-3k\pi R} \phi \tilde{q} , \quad F_{\tilde{q}} = e^{-3k\pi R} q \phi .\end{aligned}\tag{4.14}$$

Therefore, due to the wave-function in (4.13), the meson-quark coupling gets changed and become

$$e^{-6k\pi R} h^2 |\phi|^2 (|q|^2 + |\tilde{q}|^2) .\tag{4.15}$$

In the ISS vacuum, the quark vev's are as in 4d

$$q = \tilde{q}^T = \begin{pmatrix} \mu I_N \\ 0 \end{pmatrix} ,\tag{4.16}$$

Then (4.15) reproduces the light meson mode (4.13). In the SUSY vacuum in which mesons get vev's, quark masses are also redshifted by the same factor $m_q^2 = m_{\tilde{q}}^2 = \exp(-6k\pi R) h^2 |\phi|^2$. Therefore the distance in field space between the ISS and the SUSY vacuum is greatly enhanced $\Delta\phi = \exp[3k\pi RN/(N_f - N)] \Delta\phi_{ISS}$, whereas the barrier remains unchanged $V_{peak} = N_f h^2 \tilde{\mu}^4$. Therefore the bounce action S_b in the triangular approximation $S_b \sim (\Delta\phi)^4 / V_{peak}$ [15] and the lifetime of the false vacuum are accordingly increased

$$S_b \rightarrow e^{\frac{12k\pi RN}{N_f - N}} S_b .\tag{4.17}$$

However, as discussed in the previous section, a more detailed analysis of gravitational effects is needed in order to check if they are negligible. Again, if the metastable vacuum has zero energy whereas the stable vacuum has negative one, one expects the lifetime to be increased and eventually the false vacuum to become completely stable [2].

Notice that for values $h^2\tilde{\mu}^2 \sim k$ and by defining the mass scale on the IR brane with a dynamical scale $\Lambda \equiv k \exp(-k\pi R)$, we can rewrite qualitatively (4.13) in the suggestive way

$$m \sim \frac{\Lambda^3}{M_P^2}, \quad (4.18)$$

where M_P is the 4d Planck mass. It is interesting to notice the analogy between (4.18) and the scale of supersymmetry breaking in the observable sector in $\mathcal{N} = 1$ supergravity with a gaugino condensation $\langle\lambda\lambda\rangle = \Lambda^3$ in a hidden sector, coupled gravitationally with the observable one.

The models presented here can be interpreted from a holographic point of view. The metastable susy breaking can be understood in a purely four-dimensional way as arising from the infrared dynamics of a strongly coupled CFT sector, dual to the bulk geometry and the bulk fields $\Phi_{1,2}$. This CFT acts as a hidden sector, coupled to the quarks living on the UV brane. In the first example presented in this section (zero bulk mass), the light mode mediating the vacuum decay is localized on the UV brane, and from the point of view of the 4D theory it is a fundamental degree of freedom. The redshift of the mass parameter $\tilde{\mu}$ could be interpreted as the holographic version of the retrofitting discussed in [17, 19]. In the second example, in which the bulk field profile is given by eq. (4.13), the light mode is peaked on the IR brane and couples only gravitationally to the UV brane. In both cases, in the holographic 4D theory description the symmetry breaking occurs as an infrared effect, generating a hierarchy of scales like in eqs. (4.8) and (4.18)

In other types of models, in the nontrivial limit in which the boundary masses are large and the KK modes are expected to play a role in the bounce, there is no light mode anymore in the spectrum and the methods of Section 3 are needed in order to estimate the lifetime of the false vacuum. Finally, we would like to point out that there is nothing peculiar about the ISS model from the point of view of a phenomenological construction in a 5d warped space. Traditional O’Rafeartaigh models can be similarly discussed, with corresponding mass parameters and consequently scale of supersymmetry breaking redshifted to very small values. Since our main motivation was to understand the properties of the classical kink and bounce solutions, we refrain ourselves to discuss further here these applications.

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A. Standard Kink solution

Here, we remind the reader of the standard domain wall, or kink, solution. Consider a scalar field with quartic potential,

$$V(\Phi) = \frac{\lambda}{4} (\Phi^2 - v^2)^2 . \quad (\text{A.1})$$

The one-dimensional field equation,

$$\Phi''(x) = \frac{dV}{d\Phi} = \lambda\Phi(\Phi^2 - v^2) , \quad (\text{A.2})$$

with boundary conditions $\Phi(-\infty) = -v$, $\Phi(+\infty) = v$ is solved by:

$$\Phi_{kink}(x) = v \tanh \mu x, \quad \mu \equiv \sqrt{\frac{\lambda v^2}{2}} . \quad (\text{A.3})$$

This is also a solution of the first order equation:

$$\mathcal{E} \equiv \frac{(\Phi')^2}{2} - V(\Phi) = 0 . \quad (\text{A.4})$$

This can be read as the conservation of energy equation of a point particle moving in the potential $-V$ with vanishing total “energy” \mathcal{E} .

The total energy of the kink is

$$E = 2 \int_{-\infty}^{\infty} dx V(\Phi) = \frac{2\sqrt{2\lambda}}{3} v^3 = \frac{16}{3} V(0) \frac{1}{\mu} . \quad (\text{A.5})$$

B. Conserved energy

In this appendix we derive the conserved “energy,” eq. (2.35). We start from equation (2.26) which we write in the form

$$\sum_{n=1}^{\infty} a_n \partial^{2n} f - V'(f) = 0, \quad (\text{B.1})$$

where the a_n are defined by $\tan(\pi R x) = \sum_n a_n x^{2n-1}$. Next we multiply (B.1) by ∂f and use the following identity

$$\partial f \partial^{2n} f = \frac{1}{2} \partial \sum_{p=1}^{2n-1} (-1)^{p+1} \partial^p f \partial^{2n-p} f \quad (\text{B.2})$$

to get

$$\partial \left[\frac{1}{2} \sum_{n=1}^{\infty} a_n \sum_{p=1}^{2n-1} (-1)^{p+1} \partial^p f \partial^{2n-p} f - V(f) \right] = 0. \quad (\text{B.3})$$

We deduce the conserved quantity

$$\pi R \mathcal{E} = \frac{1}{2} \sum_{n=1}^{\infty} a_n \sum_{p=1}^{2n-1} (-1)^{p+1} \partial^p f \partial^{2n-p} f - V(f). \quad (\text{B.4})$$

Symbolically the sum $\sum_{p=1}^{2n-1} (-1)^{p+1} \partial^p f \partial^{2n-p} f$ can be written as

$$\frac{\partial_1 \partial_2}{\partial_1 + \partial_2} (\partial_1^{2n-1} + \partial_2^{2n-1}) f(x_1) f(x_2)|_{x_1=x_2=x}, \quad (\text{B.5})$$

where $\partial_i = \partial_{x_i}$. The first term in (B.4) can thus be put in the form

$$\frac{\partial_1 \partial_2}{\partial_1 + \partial_2} (\tan(\pi R \partial_1) + \tan(\pi R \partial_2)) f(x_1) f(x_2)|_{x_1=x_2=x} \quad (\text{B.6})$$

Now we use

$$\tan(\pi R \partial_1) + \tan(\pi R \partial_2) = [1 - \tan(\pi R \partial_1) \tan(\pi R \partial_2)] \tan(\pi R (\partial_1 + \partial_2)) \quad (\text{B.7})$$

and $(\partial_1 + \partial_2)^n f(x_1) f(x_2)|_{x_1=x_2=x} = \partial^n f^2$, which gives

$$\tan(\pi R (\partial_1 + \partial_2)) f(x_1) f(x_2)|_{x_1=x_2=x} = \tan(\pi R \partial) f^2. \quad (\text{B.8})$$

Collecting all the terms we get the final expression

$$\pi R \mathcal{E} = \frac{1}{2} \left[\frac{\tan(\pi R \partial)}{\partial} \right] [(\partial f)^2 - (\partial \tan(\pi R \partial) f)^2] - V(f). \quad (\text{B.9})$$

C. The 5D Kink near the extrema of the scalar potential

Here we analyze the behavior of the kink in flat space, close to the extrema of the potential, $f = 0, \pm v$. In particular we show that a solution with zero “energy” \mathcal{E} , i.e. satisfying eq. (2.42), cannot cross from a region where $|f| < v$ to another one where $|f| > v$.

We have already shown in Section 2.1.2 that when $f \sim v$ the solution to eq. (2.26) is exponential,

$$f \sim v \pm \eta \exp[\pm x/l_w], \quad \frac{1}{l_w} \tan\left(\frac{\pi R}{l_w}\right) = 2\lambda v^2, \quad (\text{C.1})$$

where η is a constant. One can check that the above ansatz satisfies the condition $\mathcal{E} = 0$ to lowest order in η : inserting (C.1) in (2.35) and keeping terms quadratic in η we obtain (for any choice of signs in (C.1)):

$$\begin{aligned} \mathcal{E} &= \frac{1}{2} \frac{\tan(\pi R \partial)}{\pi R \partial} \left[\left(\frac{\pm \eta}{l_w} e^{\pm x/l_w} \right)^2 - \left(\tan(\pi R/l_w) \frac{\pm \eta}{l_w} e^{\pm x/l_w} \right)^2 \right] - \frac{\lambda v^2 \eta^2}{\pi R} e^{\pm 2x/l_w} \\ &= \left\{ \frac{1}{2} \frac{\tan(2\pi R/l_w)}{2\pi R/l_w} \left[\frac{1}{l_w^2} (1 - \tan^2(\pi R/l_w)) \right] - \frac{\lambda v^2}{\pi R} \right\} \eta^2 e^{\pm 2x/l_w} \\ &= \left\{ \frac{1}{l_w} \tan\left(\frac{\pi R}{l_w}\right) - 2\lambda v^2 \right\} \frac{\eta^2 e^{\pm 2x/l_w}}{2\pi R} = 0, \end{aligned}$$

where in the last line we used the identity:

$$\tan 2z = \frac{2 \tan z}{1 - \tan^2 z}. \quad (\text{C.2})$$

Each solution of the type (C.1) approaches $\pm v$ as $|x| \rightarrow \infty$. One can ask whether it is possible for a solution to approach (and cross) $|f| = v$ at a finite value $x = x_0$. A priori, one can take :

$$f \sim v + \eta \sinh[(x - x_0)/l_w] \quad x \approx x_0, \quad (\text{C.3})$$

as a solution to the linearized kink equation (2.26) with the desired property to cross $f = v$ at $x = x_0$. However, let us compute the conserved energy for (C.3):

$$\begin{aligned} \mathcal{E} = & \frac{1}{2} \frac{\tan(\pi R \partial)}{\pi R \partial} \left[\left(\frac{1}{l_w} \cosh[(x - x_0)/l_w] \right)^2 - (\tan(\pi R \partial) \partial \sinh[(x - x_0)/l_w])^2 \right] \eta^2 \\ & - \lambda v^2 \eta^2 \sinh^2[(x - x_0)/l_w]. \end{aligned} \quad (\text{C.4})$$

We will use the following formal identity: for any differential operator $\hat{O}(\partial)$ constructed with a function $O(k)$ which has an expansion containing only even powers of k , (such as the two operators appearing in the above expression) we have:

$$\hat{O}(\partial) \sinh kx = O(k) \sinh kx, \quad \hat{O}(\partial) \cosh kx = O(k) \cosh kx. \quad (\text{C.5})$$

Using this fact, and some manipulation of the hyperbolic functions, we arrive at :

$$\mathcal{E} = \left\{ \frac{1}{4l_w^2} \left(1 + \tan^2 \left(\frac{\pi R}{l_w} \right) \right) + \frac{\lambda v^2}{2\pi R} \right\} \eta^2 > 0, \quad (\text{C.6})$$

therefore a solution that crosses $f = \pm v$ cannot have a zero value of \mathcal{E} .

On the contrary, a zero energy solution can cross $f = 0$ at some finite value of x . Close to $f = 0$ the solution of the linearized equation has now the form (see eq. (2.32)):

$$f(x) \simeq \eta \sin[(x - x_0)/l_0], \quad \tanh[\pi R/l_0] = \lambda v^2 l_0, \quad (\text{C.7})$$

Inserting this in eq. (B.9) and performing the same steps that led to eq. (C.6) we obtain:

$$\mathcal{E} = \left\{ \frac{1}{4l_0^2} \left(1 - \tanh^2 \left(\frac{\pi R}{l_0} \right) \right) + \frac{\lambda v^2}{2\pi R} \right\} \eta^2 - \frac{\lambda v^4}{4\pi R}. \quad (\text{C.8})$$

The last term is η -independent, and comes from the non-zero value of $V(f)$ at $f = 0$. For an appropriate choice of η , we can make \mathcal{E} vanish:

$$\eta^2 = \frac{\lambda v^4}{4\pi R} \left\{ \frac{1}{4l_0^2} \left(1 - \tanh^2 \left(\frac{\pi R}{l_0} \right) \right) + \frac{\lambda v^2}{2\pi R} \right\}^{-1} \Rightarrow \mathcal{E} = 0 \quad (\text{C.9})$$

Notice that this argument does not require η to be small: the validity of the linearized approximation made in eq. (C.7) holds for arbitrary η , as long as x is close enough to x_0 .

References

- [1] S. R. Coleman, Phys. Rev. D **15**, 2929 (1977) [Erratum-ibid. D **16**, 1248 (1977)].
C. G. Callan and S. R. Coleman, Phys. Rev. D **16** (1977) 1762.
- [2] S. R. Coleman and F. De Luccia, Phys. Rev. D **21** (1980) 3305.
- [3] K. Intriligator, N. Seiberg and D. Shih, JHEP **0604** (2006) 021 [arXiv:hep-th/0602239];
K. Intriligator and N. Seiberg, arXiv:hep-ph/0702069.
- [4] S. Franco and A. M. Uranga, JHEP **0606** (2006) 031 [arXiv:hep-th/0604136]; H. Ooguri and Y. Ookouchi, arXiv:hep-th/0606061 and arXiv:hep-th/0607183 ; V. Braun, E. I. Buchbinder and B. A. Ovrut, Phys. Lett. B **639** (2006) 566 [arXiv:hep-th/0606166] and arXiv:hep-th/0606241; S. Franco, I. Garcia-Etxebarria and A. M. Uranga, arXiv:hep-th/0607218; S. Forste, arXiv:hep-th/0608036; A. Amariti, L. Girardello and A. Mariotti, arXiv:hep-th/0608063 and arXiv:hep-th/0701121; I. Bena, E. Gorbato, S. Hellerman, N. Seiberg and D. Shih, arXiv:hep-th/0608157; C. Ahn, arXiv:hep-th/0608160 and arXiv:hep-th/0610025; M. Eto, K. Hashimoto and S. Terashima, arXiv:hep-th/0610042; R. Tatar and B. Wetenhall, JHEP **0702** (2007) 020 [arXiv:hep-th/0611303]; R. Argurio, M. Bertolini, S. Franco and S. Kachru, arXiv:hep-th/0610212; A. Giveon and D. Kutasov, arXiv:hep-th/0703135; I. Garcia-Etxebarria, F. Saad and A. M. Uranga, arXiv:0704.0166 [hep-th]; C. Angelantonj and E. Dudas, arXiv:0704.2553 [hep-th].
- [5] W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. **83**, 4922 (1999) [arXiv:hep-ph/9907447].
- [6] O. DeWolfe, D. Z. Freedman, S. S. Gubser and A. Karch, Phys. Rev. D **62**, 046008 (2000) [arXiv:hep-th/9909134].
- [7] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **125** (1983) 136 ; K. Akama, Lect. Notes Phys. **176** (1982) 267 [arXiv:hep-th/0001113].
- [8] S. C. Davis and S. Brechet, Phys. Rev. D **71** (2005) 104023 [arXiv:hep-ph/0503243].
- [9] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 3370 [arXiv:hep-ph/9905221].
- [10] N. Marcus, A. Sagnotti and W. Siegel, Nucl. Phys. B **224** (1983) 159; N. Arkani-Hamed, T. Gregoire and J. G. Wacker, JHEP **0203** (2002) 055 [arXiv:hep-th/0101233].
- [11] see e.g. E. A. Mirabelli and M. E. Peskin, Phys. Rev. D **58** (1998) 065002 [arXiv:hep-th/9712214] ; J. Bagger, F. Feruglio and F. Zwirner, JHEP **0202** (2002) 010 [arXiv:hep-th/0108010] ; E. Dudas, T. Gherghetta and S. Groot Nibbelink, Phys. Rev. D **70** (2004) 086012 [arXiv:hep-th/0404094].
- [12] M. Dine and J. Mason, arXiv:hep-ph/0611312 ; H. Murayama and Y. Nomura, Phys. Rev. Lett. **98** (2007) 151803 [arXiv:hep-ph/0612186] and Phys. Rev. D **75** (2007) 095011 [arXiv:hep-ph/0701231] ; R. Kitano, H. Ooguri and Y. Ookouchi, Phys. Rev. D **75** (2007) 045022 [arXiv:hep-ph/0612139] ; C. Csaki, Y. Shirman and J. Terning, arXiv:hep-ph/0612241 ; S. A. Abel and V. V. Khoze, arXiv:hep-ph/0701069.
- [13] E. Dudas, C. Papineau and S. Pokorski, JHEP **0702** (2007) 028 [arXiv:hep-th/0610297] ; H. Abe, T. Higaki, T. Kobayashi and Y. Omura, Phys. Rev. D **75** (2007) 025019 [arXiv:hep-th/0611024] ; R. Kallosh and A. Linde, JHEP **0702** (2007) 002 [arXiv:hep-th/0611183] ; O. Lebedev, V. Lowen, Y. Mambrini, H. P. Nilles and M. Ratz, JHEP **0702** (2007) 063 [arXiv:hep-ph/0612035].
- [14] T. Gherghetta and A. Pomarol, Nucl. Phys. B **586** (2000) 141 [arXiv:hep-ph/0003129].

- [15] M. J. Duncan and L. G. Jensen, Phys. Lett. B **291** (1992) 109.
- [16] N. Arkani-Hamed, M. Porrati and L. Randall, JHEP **0108** (2001) 017
[arXiv:hep-th/0012148] ; R. Rattazzi and A. Zaffaroni, JHEP **0104** (2001) 021
[arXiv:hep-th/0012248]; for a review see T. Gherghetta, arXiv:hep-ph/0601213.
- [17] M. Dine, J. L. Feng and E. Silverstein, Phys. Rev. D **74** (2006) 095012
[arXiv:hep-th/0608159].
- [18] M. Aganagic, C. Beem, J. Seo and C. Vafa, arXiv:hep-th/0610249.
- [19] F. Brummer, arXiv:0705.2153 [hep-ph].